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## Geometrical standards in shapes of avian eggs

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The original technique of description of avian eggs on the basis of the geometry of asymmetrical oval (ovoid) is suggested. Specific properties of this figure allow to create a system of 80 basic ovoid standards, each given an appropriate name, digital and letter coding, and distinct quantitative characteristics. Combining infundibular zones (blunt poles) of basic ovoids in pairs gives 80 standards of symmetric pseudo-ovoids, 44 of which are found in birds. The same procedure applied to different ovoids produces 375 standards of asymmetrical pseudo-ovoids. This totality can be divided into six groups. Use of such system of standards enables us to identify real shapes of avian eggs, to analyze relation of morphometric parameters to incubatory properties of eggs, and also to carry out comparisons and generalizations of other authors' data. Each standard is quantitatively characterized by means of indexes (namely, indices of infundibular, cloacal, and lateral zones; index of asymmetry, elongation index, complementarity index, interporal index, arc radiuses, length and diameter).

Key words: ovoid; pseudo-ovoid; indexes of egg shape; classification of shapes of avian eggs

#### Introduction

Classification of any process in nature is based on the singling out few groups with same features from total number of objects. The mentioned above can be chosen arbitrarily or by special system. In such system, the transition from one object to another is made by unified principle, on which all system is based.

From the geometric point of view the bird's egg as biological body is arranged simply. However, specificity of egg shapes in different bird species generates certain difficulties in the course of their description and classification. In field conditions, it is easy to take only two measurements directly from an egg: diameter and length. Certainly, this is not enough for full description of shape. Additional parameters can be taken either from a plane projection (blueprints, photos) (Führer-Nagy, 2002; Kostin, 1977; Myand, 1988; Romanoff, Romanoff, 1949) or by means of specially constructed devices (Preston, 1953; 1968). Digital photography and its computer processing have simplified the process of obtaining of additional parameters. There are many papers already dealing with such techniques of egg description (Anderson, 1978; Baker, 2002; Barta, Székely, 1997; Bridge et al., 2007; Makatsch, 1974; Makatsch, 1976; Monus, Barta, 2005; Mytiai, 2003; 2008; Todd, Smart, 1984). Nevertheless, the advantage of the mentioned techniques could not solve the problem related to denomination and classification of egg shapes and possibility of comparison of egg morpho-metric data published by various authors.

Literary sources show no single approach concerning names of egg shapes. In one cases they are non-informative, like "an egg with pronounced blunt and sharp ends", in others they are tautological: "ovoid type of egg shape". Certain shortcomings are inherent in geometrically determined designations (spherical, ellipsoidal, oval) or in referring to certain specific bodies (tear-shaped, pear-shaped, pegtop-shaped). Often this similarity is rather conditional, as in real eggs opposite poles may represent different figures. There is no unity also in quantity of basic figures. Different authors mark out: three (Klimov, 1993; Makatsch, 1974; 1976), four (Narushin, 2005; Preston, 1953), five (Gotman, Jablonski, 1972), eight (Walters, 1994), ten (Barta, Székely, 1997; Romanoff, Romanoff, 1949) of them (the author considered the various adjectives to indicated shapes: long, short etc.). Besides, the suggested figures are regarded out of unity and without quantitative characteristics.

Generally, modern oology is characterized by availability of huge quantity of factual materials, it has a lot of most modern mathematical methods of describing, but till this time there is no single system, which includes adequate names of egg forms and which is followed by appropriate geometrical figure and algebraic equation. The fragments of similar systems and methodic of their creation are described enough in the literature (Baker, 2002; Köller, 2000; Monus, Barta, 2005; Narushin, 2005; Preston,

1968; Preston, 1969; Todd, Smart, 1984; Frantsevich, 2017). Nevertheless, works which include aspects mentioned above are missed in completed mode.

In this connection, we have assumed the attempt of removing the mentioned problems by merging of few modern methodic in single system, which gives the opportunity to present all manifold of avian egg forms with the help of geometrical standards, which can be output from single figure (ovoid). Each of this standard has his own digital code, mode of geometrical construction and appropriate algebraic equations. This message is attempting to solve the problem.

#### Material and methods

As a basic model, the author uses a figure which is called "ovoid" or "asymmetrical oval" in descriptive geometry and engineering graphics (Fig. 1).



**Figure 1**. Generalized scheme of ovoid and variants of reading of parameters:  $O-O_3$ - centers of conjugated arcs; P,  $P_1$ - points of intersection of lateral arcs;  $B_c$ - base circle;  $I_z$ ,  $L_z$ ,  $C_z$  - infundibular, lateral and cloacal zones of ovoid and their radiuses:  $r_k$ ,  $r_k$ ,  $r_c$ , D- diameter; L - length;  $I_k$ ,  $I_c$ - infundibular and cloacal portions of length.

According to one of definitions (Cundy, Rollett, 1989; Dixon, 1991), ovoid is a flat, closed, convex, smooth curve consisting of conjugated arcs of circles of different radiuses. Characteristic features of this curve are the existence of one axis of symmetry and not less than four apexes. In real eggs, these apexes have corresponding zones: infundibular (the zone where air camera is located), cloacal (opposite, more pointed, the room for allantois) and two laterals, which are sides of interpolar zone where germ is located. On a plane projection, each of these zones is outlined by arcs of the same name, forming an ovoid upon their conjugation.

For description and classification of egg shapes the author uses two models. First – the model of composite ovoid. It is a closed curve consisting of arcs that merge seamlessly into each other, according to which all variety of shapes can be obtained by composition (combination, conjugation, faired interception) of arcs, adequate to curvature of zones of ovoid. For every possible shape the geometrical figure, which visually reflects relations of cloacal and infundibular arcs, length, and diameter was built. This model gives the opportunity graphically present all egg forms in single universal geometrical system at the expense of ovoid conversion.

Second model is polynomial. It represents the physical sense of egg, as evenly or not evenly compressed sphere. Equation of polynomial and appropriate coefficients for our database were calculated and kindly transferred to us by L.I. Francevich, his methodic presented in his work (Frantsevich, 2017).

Quantitative description of ovoids was carried out by means of seven shape indexes: the traditional elongation index  $I_{el}=L/D$ , and six indexes proposed herein:  $I_{iz}=r_i/D$ ;  $I_{cz}=r_c/D$ ;  $I_{eq}=L-(r_i+r_c)=I_{el}-(I_{1z}+I_{cz})$ ;  $I_{as}=I_{iz}-I_{cz}=(r_i-r_c)/D$ ;  $I_{com}=(r_c+I_{eq})(I_{eq}+r_i)/I_{eq}L$ , where  $I_{iz}$ ,  $I_{cz}$ ,  $I_{cz}$ ,  $I_{cz}$ ,  $I_{es}$ ,  $I_{eh}$ ,  $I_{com}$  are indexes of infundibular, cloacal and lateral zones, index of asymmetry, interpolar and complementarity indexes;  $r_i$ ,  $r_c$ ,  $r_l$  – are radiuses of arcs; L – length; D – diameter. The last index reflects the degree of balance (harmony) of infundibular ( $r_i$ ) and cloacal ( $r_c$ ) radiuses with the length of ovoid. In conformal geometry, this index is called "cross ratio" (or "vurf").

For second model for each form was adduced the polynomial equation.

Initial parameters – length (L) and diameter (D) – of real eggs were measured by vernier caliper with an accuracy of 0.1 mm. Measurements of necessary circle arc radiuses were carried out on digital photos by means of computer programs developed by B. Trotsenko and S. Shelestyuk. The program was written according to the equation smooth piecewise continuous curve (appropriate equation presented in supplement). The author expresses sincere gratitude to them.

The volume of studied material makes 16494 eggs of 800 species belonging to 20 bird orders of North-Western Palearctic.

Matter of proposed methodic is in next. As it was showed above, egg profile come out from smooth cross-over of two polar and two lateral arcs each other, in the result we gain closed loop (Fig. 2).



Figure 2. Modes of geometrical construction of the ovoid

That is what it is ovoid (asymmetrical oval). As it seen from picture the combination of arcs subject to strict parameters, inherent right figures, like for example cube, to which ovoid by its parameters closely linked. These parameters are: ri=0,5D, rl=2D; rc=L-D=1-( $\sqrt{2}/2$ )=0,293D; L=2-( $\sqrt{2}/2$ )=1,293D.

Sequentially changing the sizes of mentioned above arcs, we gain different profiles within a single system. They were proposed as standards for classification of the real forms of avail eggs. The diameter was taken as a unit, left four parameters were used by us for calculating of form indexes.

We believe that all forms, that have radius of curvature infundibular (more round) zone, that coming to the half of diameter, were marked in the group of basic ovoids. Eggs with such parameters compose 21,2% from form quantity, that was presented in our database (n=16494). Other forms, those have less radius gain the name - pseudoovoids. Those had equal radiuses curvature of the polar zones were named symmetrical, other – asymmetrical pseudoovoids (Fig.3).



Figure 3. Shapes of eggs by configuration of polar zones: a) symmetric pseudo-ovoid; b, c) asymmetric pseudo-ovoid

Symmetric and asymmetric pseudoovoids can be shown as two equal or different ovoids (Fig. 4).



Figure 4. Modes of constructing the symmetric and asymmetric pseudoovoids

As seen from the previous figures the diversity raised in area, limited by two lateral arcs, which crossed each other in points P  $\mu$  P<sub>1</sub>, that create a discrete number of forms (figure. 5).



Figure 5. Intervals and interrelation of lateral arcs with diameter and length of ovoids

Interrelation between radiuses of lateral arcs and distances between their points of intersection is expressed by the following equation:  $PP_1 = \sqrt{(4Dr_1 - D^2)}$ . Accordingly, distances equal to square roots of 2-7 correspond to the following radiuses of lateral arcs: 0.75D; 1.0D; 1.25D; 1.5D; 1.75D; 2.0D. In this way, we gained six, kind of geometrical matrixes, that were used for differentiation of forms, at the constant rate of curvature infundibular zone.

Hereby conjugation of lateral arcs is carried out by cloacal circle in such a way that its opposite to the point of conjugation part can be in various positions on the longitudinal axis of egg profile depending on diameter (Fig. 6).



**Figure 6**. Types of ovoids by configuration of cloacal zone: a) sphere-shaped; b) roundish; c) blunt; d) typical; e) drop-shaped; f) cone-shaped

Conditionally breaking this axis into intervals, we will have an opportunity to express radiuses of cloacal arcs quantitatively, through halves of length of ovoid: L-0.125; L-0.25; L-0.5; L-0.75; L-D; (L-D)/2; (L-D)/4. Hence, we obtain six types of ovoids, named according to the position of cloacal circle: sphere-shaped, roundish, blunt, typical, drop-shaped, and cone-shaped. All these shapes differ from each other by the radius of cloacal arc. In this regard, above-mentioned ovoids receive additional names: large-radius, medium-radius and small-radius. Length of ovoids vary depending on in which lateral arcs conjugation appears. It gives five additional names: short, short-cut, normal, elongated and long. Each of this form can be quantified trough three of five parameters ( $r_c$ ,  $r_l$ ,  $r_i$ , L,D) and by means of polynomial equation. Taking into account the foregoing geometrical features of ovoid, we have developed a system of 80 basic ovoids, belonging to six types.

#### **Results and discussion**

The initial point of classification of profiles standards of avial eggs were creating the single universal system of ovoids. As it was marked below, as constants were used: egg diameter, that equals one and radius of infundibylar zone equals the half of diameter. Wherein cloacal radiuses and lateral arcs remain volatile. Based on the ovoid geometry, as intervals for lateral arcs were chosen radiuses equal square roots from 2-6. Within these arcs considedred maximal, average and minimal sizes of cloacal arcs. The standards were divided into six types.

<u>The first group</u> includes forms, which are similar with sphere. They gained the name - sphere-like. Their cloacal circles have diametres within  $D>d_{c}>L-0,125D$  (Fig. 7).



Figure 7. Sphere-like ovoids

The elongation index of such eggs approaches to one:  $1.0 < l_{el} \le 1.09$ . We find such shapes in birds very seldom, about 0.2% (n=3498). They occur in orders Passeriformes, Galliformes, and Piciformes.

<u>The second type</u> includes shapes having diameters within limits of  $L-0.125D>d_c \ge L-0.375D$ . They are provisionally referred to as roundish: large-radius (6–10), medium-radius (11–15) and small-radius (16–20), with additional characteristics as short (6; 11; 16), short-cut (7; 12; 17), normal (8; 13; 18), elongated (9; 14; 19), long (10; 15; 20).

The elongation index: *1.091<lel≤1.287*. Occurrence of these shapes is near 4.0% in orders mentioned above and in Passeriformes, Galliformes, and Piciformes as well (Fig. 8).



Figure 8. Roundish ovoids

<u>The third type</u> includes shapes having diameters within:  $L-0.375D>d_c \ge L-0.625D$  (Fig.9) and is represented with shapes referred to as blunt ovoids. Each of these form, as in previous case, can be divided into large-, medium- and small-radius shapes (21–25; 26–31; 32–36), and respectively: short (21; 26; 32), short-cut (22; 27; 33), normal (23; 28; 34), elongated (24; 30; 35) and long (25; 31; 36). The elongation index of these falls within limits:  $1.146 < I_{el} \le 1.4$ . The mentioned shapes occur in percentage up to 4.9% in representatives of such orders as Charadriiformes, Falconiformes, Galliformes, Passeriformes, Piciformes, and in small numbers in Coraciiformes, and Gruiformes.

<u>The fourth type</u> includes shapes in which cloacal circles fall within limits of L-0.625D>d<sub>c</sub>≥L-0.75D (Fig. 10). They are referred to as typical ovoids: large-radius (36–40), medium-radius (41–45) and small-radius (46–50); and by length: short (36; 41; 46), short-cut (37; 42; 47), normal (38; 43; 48), elongated (39; 44; 49) and long (40; 45; 50). The elongation index of these eggs falls within limits:  $1.146 < I_{el} \le 1.4$ . Such shapes occur in percentage up to 41.2% in such orders as Anseriformes, Charadriiformes, Falconiformes, Galliformes, Gruiformes, Passeriformes, and Piciformes.



Figure 9. Blunt ovoids

<u>The fifth type:</u> cloacal radiuses fall within limits:  $L-0.75D > d_c \ge L-1.125D$  (Fig. 11). These shapes are referred to as drop-shape ovoids. They include large-radius (51–55), medium-radius (56–60) and small-radius (61–65) eggs. By length: short (51; 56; 61), short-cut (52; 57; 62), normal (53; 58; 63), elongated (54; 59; 64), long (55; 60; 65). The elongation index of these eggs lies in the limits:  $1.323 < I_{el} \le 1.643$ . These shapes make up 48.2% in the orders Charadriiformes and Passeriformes.

<u>The sixth type</u>: cloacal circles in the limits  $L-0.125D>d_c \ge L-2.0D$  (Fig. 12). They are referred to as cone-shaped: large-radius (66–70), medium-radius (71–75) and small-radius (76–80); and by length: short (66; 71; 76), short-cut (67; 72; 77) normal (68; 73; 78), elongated (69; 74; 79) and long (70; 75; 80). The elongation index falls within the limits:  $1.449 < l_{el} \le 1.745$ . These shapes make up 1.5% only in Charadriiformes.



Figure 10. Typical ovoids



Figure 11. Drop-shaped ovoids



Figure 12. Cone-shaped ovoids

The above-mentioned geometrical standards have fixed (individual) characteristics, expressed in the form of cloacal and lateral zones as well as in the form of indexes of elongation, complementarity, asymmetry and interporal. Thus, mentioned above 80 geometric forms obtain position number, names, and quantitative adjectives, which we propose as the basics (table 1).

N₂	Standard name	lcz	llz	lel	lcom	las	lez
		Spher	e-like ovoid	ls:			
1	short	0,480	0,75	1,083	3,152	0,020	0,103
2	short-cut	0,487	1,0	1,098	2,987	0,013	0,112
3	normal	0,490	1,25	1,111	2,822	0,010	0,12
4	elongated	0,493	1,5	1,115	2,803	0,007	0,123
5	long	0,494	1,75	1,116	2,814	0,006	0,12
		Rour	ndish ovoid	S			
Large	e-radius						
6	short	0,478	0,75	1,091	2,806	0,022	0,12
7	short-cut	0,488	1,0	1,108	2,826	0,012	0,12
8	normal	0,490	1,25	1,112	2,797	0,010	0,12
9	elongated	0,491	1,5	1,140	2,439	0,009	0,15
10	long	0,493	1,75	1,143	2,432	0.007	0,15

Ukrainian Journal of Ecology, 7(3), 2017

Medi	um-radius							
11	short	0,437	0,75	1,125	2,029	0,063	0,189	
12	short-cut	0,459	1,0	1,169	1,935	0,041	0,210	
13	normal	0,468	1,25	1,190	1,883	0,032	0,223	
14	elongated	0.473	1.50	1.200	1.868	0.027	0.227	
15	long	0.477	1 75	1 207	1 856	0.023	0.231	
Smal	Iradius	0,477	1,75	1,207	1,050	0,025	0,231	
Sinai	-i autus							
16	short	0,387	0,75	1,151	1,637	0,113	0,264	
17	short-cut	0,423	1,0	1,218	1,589	0,077	0,295	
18	normal	0.441	1.25	1.256	1.556	0.059	0.316	
19	elongated	0 4 4 9	, 1 5	1 271	1 549	0.051	0 322	
20	long	0,445	1 75	1,271	1,545	0,031	0,322	
20	long	0,450	1,75	1,207	1,555	0,044	0,551	
		BI	unt ovoids					
Large	e-radius							
21	short	0 382	0 75	1 146	1 629	0 1 1 8	0 265	
21	short-cut	0,302	1.0	1,140	1,625	0,094	0,203	
22		0,410	1,0	1,210	1,505	0,064	0,502	
23	normai	0,433	1,25	1,266	1,512	0,067	0,334	
24	elongated	0,444	1,5	1,284	1,507	0,056	0,341	
25	long	0,452	1,75	1,301	1,498	0,048	0,349	
Medi	um-radius							
26	short	0,335	0,75	1,167	1,432	0,165	0,332	
27	short-cut	0.375	1.0	1,250	1,400	0.125	0.375	
28	normal	0,403	1 25	1 306	1 3 8 3	0.097	0,403	
20	alangatad	0,403	1,25	1,500	1,505	0,097	0,403	
29	elongated	0,417	1,5	1,333	1,375	0,083	0,417	
30	long	0,430	1,75	1,359	1,368	0,070	0,430	
Smal	l raduis							
31	short	0,285	0,75	1,177	1,309	0,215	0,392	
32	short-cut	0,342	1,0	1,275	1,310	0,158	0,433	
33	normal	0,371	1,25	1,334	1,300	0,129	0,463	
34	elongated	0,389	1.5	1.379	1.288	0.111	0,490	
35	long	0.408	1.75	1,400	1,296	0.092	0.492	
00		Tvpi	cal ovoids	.,	.,	0,002	0,.52	
Large	-radius							
36	short	0 269	0.75	1 1 6 9	1 287	0 231	0 401	
37	short-cut	0,209	1.0	1,105	1,207	0,291	0,401	
20	normal	0,303	1,0	1,205	1,250	0,159	0,400	
20	olongated	0,342	1,25	1,342	1,255	0,138	0,500	
40	long	0,372	1,5	1,309	1,230	0,120	0,518	
40 Madi		0,565	1,75	1,427	1,247	0,117	0,544	
ivieui	um-radius	0.217	0.75	1 1 0 0	1 100	0.202	0.467	
41	SHOL	0,217	0,75	1,183	1,196	0,283	0,467	
42	snort-cut	0,276	1,0	1,299	1,203	0,224	0,524	
43	normal	0,312	1,25	1,374	1,202	0,188	0,562	
44	elongated	0,340	1,5	1,430	1,201	0,160	0,590	
45	long	0,356	1,75	1,146	1,534	0,144	0,606	
Smal	l-radius							
46	short	0,156	0,75	1,191	1,122	0,344	0,535	
47	short-cut	0,223	1,0	1,316	1,142	0,277	0,594	
48	normal	0,264	1,25	1,399	1,149	0,236	0,635	
49	elongated	0,307	1,5	1,451	1,164	0,193	0,644	
50	long	0,338	1,75	1,482	1,177	0,162	0,645	
		Drop-sł	naped ovoic	ls				
Large-radius								
51	short	0,212	1,0	1,323	1,131	0,288	0,612	
52	short-cut	0,254	1,25	1,408	1,138	0,246	0,655	
53	normal	0,282	1,5	1,474	1,138	0,218	0,693	
54	elongated	0,305	1,75	1,532	1,137	0,195	0,728	
55	long	0,332	, 2,0	1,555	1,147	0,168	0,724	
Medi	um-radius	, = = =		,'	,	,	,	
56	short	0,169	1,0	1,337	1,094	0,331	0,669	
-			· -		'	,		

67	chart cut	0 212	1 25	1 426	1 1 0 5		0 710		
57	short-cut	0,215	1,25	1,420	1,105	0,207	0,715		
58	normal	0,250	1,5	1,500	1,111	0,250	0,750		
59	elongated	0,277	1,75	1,553	1,115	0,223	0,777		
60	long	0,300	2,0	1,600	1,117	0,200	0,800		
Sma	ll-radius								
61	short	0,122	1,0	1,345	1,063	0,378	0,723		
62	short-cut	0,175	1,25	1,445	1,079	0,325	0,770		
63	normal	0,212	1,5	1,516	1,087	0,288	0,804		
64	elongated	0,249	1,75	1,578	1,095	0,251	0,829		
65	long	0,259	2,0	1,643	1,089	0,241	0,884		
	Cone-shaped ovoids								
Larg	e-radius		•						
66	short	0,159	1,0	1,449	1,069	0,341	0,790		
67	short-cut	0,189	1,25	1,522	1,074	0,311	0,834		
68	normal	0,222	1,5	1,603	1,078	0,278	0,882		
69	elongated	0,249	1,75	1,643	1,085	0,251	0,894		
70	long	0,150	2,0	1,541	1,055	0,350	0,891		
Medium-radius			,	,	,	,			
71	short	0,185	1,25	1,618	1,061	0,315	0,933		
72	short-cut	0.213	1, 5	1.667	1,067	0.287	0.955		
73	normal	0,147	1,75	1,541	1,053	0,353	0,895		
74	elongated	0,180	2.0	1.621	1.059	0.320	0.942		
75	long	0.209	2.0	1.667	1.065	0.291	0.959		
Sma	ll-radius	-,	_,-	.,	.,	-,	-,		
76	short	0.139	1.5	1.649	1.042	0.361	1.011		
77	short-cut	0 171	1 75	1 705	1 048	0 329	1 035		
78	normal	0 157	1 75	1 720	1 043	0 343	1.063		
79	elongated	0.125	20	1 7/2	1 032	0,345	1 1 1 0		
20	long	0,125	2,0	1,743 1 775	1,032	0,373	1,113		
00	long	0,110	2,0	1,740	1,029	0,304	1,129		

These standards enable to describe more than 20% of real egg shapes (n=16494), as well as for standards formation for other forms. Latter were obtained by means of combining (composing) of infundibular zones (blunt poles) of basic ovoids. The resulting figures were called "pseudo-ovoids", as their infundibular radius is less than half of diameter, i.e. differs from that in basic ovoids. Combining different basic ovoids we obtain standards of asymmetrical pseudo-ovoids. Combinations of identical ovoids produce the set of symmetrical pseudo-ovoids (Fig. 13).



Figure 13. Geometrical standards of symmetrical pseudo-ovoids

Hereby it is necessary to admit, that there are no eggs with absolutely equal radiuses of polar zones exist in nature. Hence, we propose to include shapes whose asymmetry index doesn't exceed 0.05 to this category. Symmetrical eggs occur in birds more seldom (up to 5.4%, n=16494), than in the rest of animals, say in reptiles. Among the most considerable reasons for that we distinguish three: a) incompact clutch; b) excessive rolling asunder of eggs; and c) inability of fixation of blastodisk towards the source of heating. Symmetrical pseudo-ovoids occur in small numbers in different orders except Gaviiformes and Charadriiformes.

Combination of 80 identical basic ovoids gives us 80 theoretically possible symmetrical pseudo-ovoids. The birds have less nimmber of such forms. Extreme, i.e. sphere-shaped, and very long or very pointed eggs are not represented in birds, although in other animal groups they are normally widespread. Our database (n=16494) shows, that 44 here suggested standards correspond to real egg shapes

As in basic ovoids, the names in this case are created by adding the definition "symmetrical pseudo-ovoid". Digital code represents binary ordinal number of basic ovoid, e.g. "blunt large-radius short-cut symmetrical pseudo-ovoid (22.22)." The majority of eggs belong to asymmetrical pseudo-ovoids (73.4%).

Their geometrical standards are obtained by combination of different basic ovoids. Shape denominations at this approach come out very long, since they involve complex names of two different basic ovoids. So we chose more simple method.

As stated above, in asymmetrical pseudo-ovoids arc radiuses of the infundibular zone are smaller than *0.5D* but always more than cloacal arc radiuses. Therefore, conjugating infundibular arcs of different radiuses with 80 basic ovoids we obtain the totality of standards of asymmetrical pseudo-ovoids.

The analysis of our oological database has shown, that infundibular arc radiuses of the discussed egg type vary within limits from *0.285D* to *0.491D*. All this totality we provisionally divide into six groups (Fig. 14).



Figure 14. Distribution of asymmetrical pseudo-ovoids by infundibular zone radiuses

Considering that shapes, making up each group, differ only by infundibular arc radiuses, their denominations are composed of names of basic ovoids with addition of group number. The code consists of the combination of digits reflecting the number of basic ovoid and the number of the group, e.g.: "blunt large-radius short-cut pseudo-ovoid of the group two (22.2)". The number of standards in each group may vary. It decreases as far as infundibular and cloacal arc radiuses coincide. Let's examine this in detail.

<u>The first group</u> includes 75 standards, in which the infundibular radius falls within the limits of  $0.491D>r_{r}>0.474D$ . This group is the closest to basic ovoids (Fig. 15). Such eggs make up 46.24% of asymmetrical pseudo-ovoids (n=12104). They are the most common in Charadriiformes and Passeriformes. They occur in Falconiformes, Galliformes, Gruiformes and Piciformes as well. The elongation index of such eggs lies within the limits:  $1.125 < l_{el} \le 1.798$ .



Figure 15. Asymmetrical pseudo-ovoids of the first group

<u>The second group</u> (Fig. 16) includes 68 standards with infundibular radiuses 0.474D > r > 0.456D. The elongation index of such eggs lies within the limits:  $1.129 < I_{el} \le 1.803$ .

276



Figure 16. Asymmetrical pseudo-ovoids of the second group

Such eggs make up 28.43 % of asymmetrical pseudo-ovoids. Most of them belong to the same orders as the previous group. They appear as well in Anseriformes, Apodiformes, Ciconiiformes, Coraciiformes, Cuculiformes, Strigiformes, and Upupiformes. The third group (Fig. 17) includes 67 standards whose infundibular radiuses lie within the limits of  $0.456D > r_i > 0.437D$ . The elongation index of such eggs lies within the limits:  $1.134 < I_{el} \le 1.786$ .



Figure 17. Asymmetrical pseudo-ovoids of the third group

Such shapes make up 12.34%. The maximal number are found in Anseriformes, Falconiformes, Galliformes, Gruiformes, Passeriformes, Piciformes, Strigiformes, and Upupiformes. They are also found in Apodiformes, Ciconiiformes, Coraciiformes, Cuculiformes, emerge in Caprimulgiformes, Columbiformes, Gaviiformes, Podicipediformes, Pelecaniformes, Procellariiformes, and decrease in Charadriiformes.

<u>The fourth group</u> (Fig. 18) is represented by 60 standards, in which the infundibular radius falls within the limits of  $0.437D > r_i > 0.419D$ . The elongation index:  $1.150 < l_{el} \le 1.806$ .



Figure 18. Asymmetrical pseudo-ovoids of the fourth group

Such shapes make up 7.54%. The distribution in avian orders is quite similar to the third group.

<u>The fifth group</u> counts 55 standards having the infundibular radius within the limits of *0.419D>r≥0.401D*. The elongation index of these eggs lies within the limits: *1.153<lel≤1.672*. Such shapes make up 3.44%. The maximal number of them occur in Anseriformes, Ciconiiformes Pelecaniformes, Podicipediformes and Gruiformes. In other orders (Falconiformes, Galliformes, Passeriformes, Columbiformes Piciformes, Strigiformes, and Upupiformes) their number is equally small. They emerge in Struthioniformes (Fig. 19).



Figure 19. Asymmetrical pseudo-ovoids of the fifth group

<u>The sixth group</u> (Figure 20) is represented by 50 standards having the infundibular radius:  $0.409D > r_i \ge 0.285D$ . The elongation index:  $1.161 < l_{el} \le 2.0$ . The share of these shapes is 2.0 %.

The group is represented basically by Anseriformes, Ciconiiformes, Podicipediformes, and Pelecaniformes. Such orders as Caprimulgiformes, Falconiformes, Gaviiformes, Gruiformes, and Passeriformes are represented equally insignificantly, the rest (Caprimulgiformes, Charadriiformes, Galliformes, Piciformes, Strigiformes, and Upupiformes) – by single specimens.

280



Figure 20. Asymmetrical pseudo-ovoids of the sixth group

The abovementioned number of standards (n=375) is deduced only by mean values of infundibular radius in six groups of asymmetrical pseudo-ovoids. Using the same scheme, while taking into account minimums and maximums, we obtain 750 standards more.

Resuming the foregoing, we must admit that the proposed system of avian egg shape standards opens several important perspectives for oological research. Giving an appropriate name and quantitative expressions to egg shapes by comparison of their photographs with geometrical standards enables to associate important biological information with any avian egg. The simplicity of the technics provides its wide applicability in different levels: visual and electronic. In the last case, the use either of existent or specially developed programs is possible. The unification and coordination of oological works of ornithologists opens new horizons for wide-ranging generalizations and creation of global databases.

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