MATHEMATICAL MODEL OF BIRD SPECIES IDENTIFYING:
IMPLICATION OF RADAR DATA PROCESSING

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Here we presented the mathematical model for the selection of birds from radar data with further possibility to identify the species. The essential background was elaborated based on the parameters could be registered by radar and the mathematical processing of database consisted of known bird species features. We suggested to use some parameters, like bird length and body mass (calculated from radar cross-section), wingspan and wingbeat frequencies, that could be also obtained from radar data. We have concluded, that the suggested method could be used in identifying an unknown species of birds from results of the measured parameters of observed unknown individual. The body mass, length, wingspan, flight speed and other bird characteristics could be used as parameters, measured by visual survey. The use of a mathematical model allows to increase the range of observations and automate the processing of the experimental data obtained with the use of modern methods (radiotelemetry, bioacoustics, radar, etc.).

Key words: mathematical model, birds, identification, radar.

INTRODUCTION

Formation of the ornithology as a science was started from the use of classic survey method. Limitations of this method is associated with a relatively small range of bird registration. The use of technical means allowing to push the observation limits and define the species more precisely. Since now the ornithologists widely used radiotracking, radiotelemetry, bioacoustics, radar and other technical methods in order to automate the observations over the birds. In this report we referred to radar signals and proposed the methods of recognizing the echoes from birds and methods of constructing vector fields of their movement. This paper is devoted to mathematical modeling of identifying bird species on the basis of field data.

At present, the radar is one of the most powerful tools available to describe the movement of birds in three-dimensional space and its use has been well documented (eg, Lack and Watley, 1945, Eastwood, 1967, Bruderer 1997, Gauthereaux & Belser 2003). Conventional radar uses short pulses of high-power radio waves and can detect an "echo" of the weaker echoes that bounce off objects. Consequently, it can be used in total darkness and in daylight, overcoming many of the limitations associated with visual observations. However, the transmitted electromagnetic pulses have no effect on migratory birds, in contrast to devices that involve the use of strong light sources (Bruderer et al., 1999). A weather radar equipment (Koistine, 2000) and the usual regular maritime navigation radar (Harmata et al., 2003) can be used to great effect to monitor bird movements due to significant reflection coefficient of the flying birds that used in the study of WPP impact on birds (Cooper 1996, Gauthereaux 1996, Johnson et al., 2002, Kahlert et al, 2002).

There are quite a large number of different types of radars and designs, which differ in their abilities to record reflections from birds to determine the height of their flight, movement speed. One of the first reviews of different pulse radars used for ornithological research, was held in (Eastwood, 1972), a comparative review of performance...
and features radar - articles (Bruderer, Stegender, 1981; Bruderer, 1987; Buurma, 1995). The ideal for the detection of birds is 5-cm radar bird detection by using longer wavelength is problematic because these waves suppress small goals; at shorter wavelengths increases the ability to display small targets, such as insects. To highlight reflections from birds on other objects of different technical solutions are applied. Circular polarization suppresses signals from the small round objects (raindrops), an indicator target movement selects objects with low radial velocity and precise time control reduces the appearance of fine goals while reducing the viewing angle. The rotating beam radars, such as aviation radars, display information on the horizontal distribution and the direction of migration (Alfa, 1995), displays a pulse volume. When using radar data often occurs rediscount high flying birds, because these radars are accurate approximation receive information about flight altitudes, but do not define the horizontal direction and placement of the birds flown. Often, the vertical and horizontal scanning radars are combined in order to obtain three-dimensional image. Modern military radars avoid the pitfalls of fan due to the existence of several rays, combined into one (Buurma, 1995). With their help, get three target coordinates. But if they are used for scanning large spatial volume, their angular resolution and accuracy is limited.

Radars with one beam receive such information by sequential scanning. Meteorologists use such radars to establish the size and intensity of the storm fronts. The main disadvantage of weather radar in terms of ornithological research - their pulse duration, leading to an increase in volume, especially over long distances. Thus, many birds can be combined into a single echo. Often used and tracking radars (Cooper et al., 1991). Such radars are informed by a fixed vertical beam located at a certain angle (Bruderer, Steigender, 1972). If the antenna is movable, the vertical beam is used for scanning perpendicular to the main direction of migration (Bruderer, 1982) or a conical scan different angles - for obtaining information about the spatial distribution of birds over the hemisphere within the radar (Bruderer, 1984). Another disadvantage of this type - the limitations of the small viewing angles. The main problem, according to many bird watchers - what counts reflected signals obtained in such a beam can not serve as objective information number and distribution of birds.

Tracking radar allows to obtain a three-dimensional position of the bird in space by a conical beam with a review of one of its small dispersion around the optical axis of the antenna or by four slightly divergent beams (monopulse radars). In all cases, information is displayed about the birds flying in the space. Writing the fluctuations of the signal, you can obtain information about the nature of flapping wings of a bird, when it comes to a single target. Using a ball-pilot allows you to add wind data information about the height of bird flight.

A number of studies (Gauthreaux, 1994; Larkin, 1994) recognized the importance of meteorological radars of new generation to prevent bird collisions with aircraft. In the US it is considered the most appropriate type of radar WSR-88D / 98D (NEXRAD). Main area of research - the use of variables to determine the polarization of birds and other filtering signals. It is planned to equip the radar so that he could send at a time horizontal and vertical polarization waves. The main variables that can be used to determine the size of the birds: differentiation phase backscatter differentiation, the correlation coefficient between the vertical and horizontal polarization data.

In some works (Eastwood, 1967; Konrad et al, 1968) provides information on the magnitude of the effective scattering surface (ESS) for a variety of bird species, obtained by hanging them on a thread. In the Soviet Union the bird ESS diagrams were obtained in an anechoic chamber (Ganya et al., 1991).

Unfortunately, certain difficulties are small birds, as in a 3-cm radar insects capable of producing large values similar to those of small birds. Values radar cross sections 10 may vary in time, depending on the position of the bird relative to the radar or aspect (Bruderer, Steigender, 1972; Eastwood, 1967). Further variations are caused by flapping wings, which increase the value of 10 times the average or reduced to almost zero at frequencies of 2-24 Hz during flapping flight of European bird species, staying on average values when the pauses between strokes.

In studies Bruderer and Liechti (1995), all recorded birds were divided into groups in accordance with the fluctuation of the signal that determines the nature of flapping wings. The following classes have been identified:
- Continuous sweeps, from 5 to 9 Hz (mainly large waders and waterfowl);
- Continuous sweeps, faster 9 Hz (small waders and waterfowl);
- Intermittent sweeps, slower 12 Hz (large passerines);
- Intermittent sweeps, faster 12 Hz (small passerines).

To confirm the results using an infrared video camera with high resolution. The data indicate that the operating width of the beam is higher than nominal, especially at small distances (Bruderer, Liechti, 1995; Bruderer et al, 1995).

Buurma (1995), along with the radar signal processing program applied motion analysis program that connects respective echo 10 from rotation. According to the direction, velocity and reflectance, the clusters corresponding signals are analyzed in accordance with the algorithm of the program are combined as the motion vector birds. The final presentation of information includes the geographic coordinates, the size of the echo and the average reflectance, direction, speed, variation for all data values and the number of echoes. To analyze the direction vectors of migratory birds is widely used non-parametric analysis of variance and Rayleigh-test (Batschelet, 1985).

Maximum power output of marine radar that is measured in kilowatts, affects the maximum range target.
detection. More powerful radars often have large engines, which means that larger antenna may be used to improve the registration of birds.

In practice, in a large range radar measurements, which means that the detection target range depends on the antenna is greater than the output power peaks. However, zoologists and ornithologists tend to use smaller portable radars with output power of 3 kW (Williams et al, 1972; Peckford & Taylor, 2008; Harmata et al, 2003) to 50 kW (Gauthreaux, 1970). The most commonly used range of 10 kW (Peckford & Taylor, 2008; Harmata et al, 2003; Tulp et al, 1999; Petersen et al, 2006.) and 25 kW (Desholm, 2003; Kahler, et al, 2003; Hupop et al, 2006). Within this range, we believe that the doubling of peak output power only increases target detection range of about 19%.

Bruderer (1997) argues that small birds identified peaks at wavelengths of 3.8-15 cm, which form the C band (3.8-7.5 cm) and lower S band (7.5-15 cm). However, even small wavelengths X radar (2.5-3.75 cm) better detect small targets, such as insects (Bruderer, 1997). Signals of longer wavelengths, on the other hand, depends less from the effects of precipitation (Richardson, 1978) and may considered the best biological observation target (poultry) under rain or snow (Larkin, Eisenberg, 1978). In our opinion, the important point is that the probability of detecting small targets of great interest to ornithologists, much lower than when using longer wavelengths (S range), and this requires further field research. Marine X radars emit electromagnetic pulses of fixed length wavelength of about 3.2 cm. marine radar is typically used as a relatively inexpensive solution algorithm based on a super-heterodyne (Richardson, 1978).

For example, radar Furuno have an initial "optimization" installation, which is installed for the purposes of tracking system.

Normal horizontal marine radar makes it possible to accurately display the trajectory of flying birds or flocks of birds in space and time (Desholm 2003). Vertically mounted radar scanner can supplement the data by measuring the height of bird flight. Using more sophisticated tracking radar could provide information from a single bird or group of birds (Alerstam & Gudmundsson 1999), combining the measurements of their movements in space (including flight altitudes) with a limited ability to determine the species. Although the radar can not identify the species of birds, it is possible to identify bird taxonomic groups based on flight speed (for example, Larkin, Thompson 1980, Larkin 1991, Evans, Driickamer, 1994, Bruderer & Boldt 2001) or wingbeat frequency (for example, Renevey 1981).

Weather radar (Gauthreaux and Belser 2003), and air traffic control radar (Beason 1978, 1980; Gauthreaux 1991; Troxel et al 2001, 2002) were used to monitor the movement of migratory birds. Marine radars less power used for basic and applied research tasks in places far from large radars, where large radar can not be used because of local ground, as well as in areas that require higher resolution to track small movements flocks or single birds.

Certain methods for collecting radar data were used in the past, including the use of photographing screen (Richardson, 1978; Harmata et al, 2003; Matsyura, 2005) and capture by the software information from the screen and then converting it into a video. However, to get the maximum amount of information from the radar signal, it is necessary to use a specialized analog to digital converter connected to a computer. Such converters have been made by researchers (e.g. VSU), available as commercial products available under the name of digital board radar. They work by using four types of analog signals from the radar:

(1) "video", which is the analog signal voltage is the output signal of the echo from the back to the radar targets.
(2) trigger or trigger pulse which accurately indicates the time when a radar pulse reflected radar to provide a reference point for synchronization of reflections from targets.
(3) the direction (or impulse installation azimuth (the ARP), which is celebrated every time the radar has completed 360 degrees and starts the cycle again.
(4) the course (or momentum changes azimuth (ACP), which has a relatively fixed angular velocity by the rotation of the antenna, and allow to execute correction changes antenna speed turns the antenna relative voltage change and wind power.

Capture and output signal processing allows the user to get more information about the biological order than is typically displayed on a standard radar screen PPI.

Typical commercial radar display is able to provide only seven levels of intensity of the echo, whereas commercial digital map models radar can provide greater intensity than 4096 levels (12 bits) with the same signal. We did an analysis of radar digital boards available from several manufacturers, including Sigma S6 map of Rutter Technologies, (St. John's, Newfoundland, Canada), XIR3000 production Russel Technologies, (North Vancouver, British Columbia, Canada).

For our ornithological tasks, taking into account the integration with Furuno and Radr program, we believe that the most appropriate is to use a card USPR-1Ettus Research, USA. The use of this card does not require special software in conjunction with the device capture the raw video from the radar screen and will provide unique opportunities for processing and interpretation of signals from birds.

Although ornithological radar system supporting post-processing of the signals, providing opportunities for data storage and management, basic radar design supports and operational use of data when using skilled operators.
radar monitor provides an unmodified form of radar data, including goals and noise. Most radar systems capable of maintaining the history of the images that can be used as a simple tool tracking purposes.

On the monitor, the computer recorded data already processed, as a rule, the card or other geo-referenced display. The processed data are continuously updated and the processor can show the coordinates of targets and tracks - move with time goals. This display provides the operator with information that can be used directly for operational purposes bird.

Typically, aircraft radar, equipped to track individual birds or bird group registering them at a distance of 20 km from the radar. As a rule, they are based on commercial, ready-made marine radars equipped with a special antenna and a transceiver that provides reliable reception of signals, which are processed by a processor avian radar signals. Modern radar systems provide continuous, throughout the day and night, in all weathers, automatic detection, tracking, localization in the Earth's coordinates, and early warning of bird flight. They can be part of a network of radars, working together to increase the coverage of a particular location or to provide data for local, regional, national or continental monitoring (Weber et al. 2005).

Commercial marine radars are available in 2-licensed bands: X-band and S-band. Release of the X-band radar 25 times the number of S-band radar (Briggs, 2004). Marine radars operating in the S-band has a wavelength of about 10 cm and a frequency of approximately 3 GHz.

An argument in favor of using radar X-band radar is usually large section of birds compared to S-band radar (Briggs 2004). Radar can not accurately measure the dimensions of the birds, even within the controlled trials (Edwards and Houghton 1959), but it can be used to provide a rough estimate of the size of the birds (Nohara et al. 2011).

Modern ornithological radar system designed to monitor the birds and provide information about their location, speed and course (Nohara et al. 2005). Techniques that allow to characterize the types of in-flight on the basis of their signal amplitude or frequency of flapping wings, are still in the development stage (Zuagg et al. 2008). Fluctuations in the amplitude of the radar signal is a combination of the signal from the moving wings birds and breathing in conjunction with the changing aspect of the bird to the radar. The resulting amplitude variations do not always accurately reflect the flapping wing frequency that can be used to identify the target as a single species or group of species of birds.

The technology for studying and monitoring bird movements using radar has undergone dramatic changes over the last 10 years, when digital computers and software signal processing software have been developed to perform this task. One result of these changes is that the digital radars generate huge amounts of data that can be used to develop programs for the automation of the data analysis. The ability to distinguish between target types (birds, insects, or aircraft), to determine the species of birds, as well as determine the size of the pack are the important goals of future research and development of ornithological radar.

Although we can not expect that it is possible to classify the bird species, a large amount of information collected ornithological radar will perform bird classification by taxonomic groups. The classification of birds based on their echo on the taxonomic groups on the basis of radar tracks performance can be significantly improved by supplementing the knowledge of the species composition of birds in the region. Currently, the speed of flight - this is the main characteristic that can be used to isolate groups of birds (Bruderer, Boldt, 2001, Alerstam et al. 2007). Correlating visual surveillance with radar tracks, one can define the local speed of certain types during the day. It is known that birds use the lower optimum flight speed for local flights, compared with migration (Hedenström and Alerstam 1995). These differences must be considered when using velocity to determine the bird species using radar tracks.

The second adjustment is the amendment of the track speed to wind speed and direction. Radar data can only be used to calculate the speed of flight of the bird relative to the ground, which is under the influence of wind speed. During migration, especially poultry, usually fly mainly downwind, resulting in birds speed relative to the earth is considerably higher than the birds in the air flow rate (Richardson, 1991).

At the time of the change of the reflected signal intensity can only be used for the approximate determination of the size and the risk of a bird (Nohara et al. 2011). radar target size gives information about the identity of target or reflection from a flock of birds. you need to use the information about the seasonal changes in the behavior of birds and species composition of birds at this time of the year to improve the classification of birds. Bird radar could be used to establish the temporal and spatial features of the migration patterns of birds moving between resting, feeding and ponds. These models are particularly important for the management of key species of birds in places such as fish ponds, airports, landfills, farm ecosystems (Klope et al. 2009). Knowledge of these laws can be used by managers when planning changes in the environment or in the analysis of the degree of influence of altered landscapes on indicator species.

Radar is used to actively control nocturnal bird migration, its intensity, timing and characteristics of the flight of different species or taxonomic groups of birds that can be supplemented by the influence of weather conditions on particular flights. In addition to speed and altitude, estimated direction (tracks), which can be classified according...
to their speed (representative of the taxonomic groups), and from these data calculated mean vector for each group (Mardia 1972, Batschelet 1981). Such information may identify specific strategies of migratory birds. For example, Gauthreaux (1991) reported that birds that migrate ahead of the cold front, differed significantly in their altitudinal distribution of birds migrating behind him. High resolution ornithological radar will allow researchers to study the effect of weather conditions on the route of individual birds in addition to the effects of these conditions on the entire population.

In principle, a new step in usage of marine radar is the vertical mode, that is, perpendicular to the ground. This will enable to obtain data on altitude of bird distribution in space and allow the construction of zone and corridors with minimum and maximum concentration of birds in space. As the radar device is mobile there is possibility to place it in the direction perpendicular to the expected direction of flight of migratory birds. This maximizes the chances of recording each group of birds as a single track.

**METHODS**

The effectiveness of ornithological research provided by the classical survey method or technical means determined by the distance to the observed object, weather conditions, precision of used instruments and other circumstances, which together create an interference that impede to obtain reliable conclusions. The main characteristics of bird species, that could be obtained during field observations are: the geometrical dimensions (length and width of the bird, wingspan, etc.); the number of wing flaps per minute; airspeed; body weight.

The accuracy of the measured parameters depends on the distance from the object, performance accuracy of instruments and weather conditions, namely: air transparency (rain, snow, fog); wind speed; wind direction; atmospheric pressure; air humidity; air temperature.

Mathematical model of bird species \( W \) observed in the real-time is associated with the accuracy of measurement, weather and other conditions, and can be represented as:

\[
W = W (U_1, U_2, \ldots, U_n, V), \quad (1)
\]

where \( W = 1, 2, \ldots, m \); \( U_1, U_2, \ldots, U_n \) - measurable parameters that form the set \( U = U \{U_k\}_{k=1}^m \), characterizing the bird species; \( V \) - noise associated with the inaccuracy of measurements.

Each bird species \( W \) is determined by set of properties \( U_{0i}, U_{02}, \ldots, U_{0n} \), which combined into a set \( U^0 = \{U^0 \}_{k=1}^m \). The reference sets for all species \( W^i \) form a standard set \( U^0 = U^0 \{U^0 \} \). Since measurement results \( U_1, U_2, \ldots, U_n \) generally do not coincide with the reference parameters \( U_{01}, U_{02}, \ldots, U_{0n} \) of studied species, then there is some chance of error identifying. Therefore, the function (1) must be considered in terms of probability. Our task was to solve the following problem:

Supposed we registered the unknown bird species with parameters \( U_1, U_2, \ldots, U_n \) during the field observations. The task is to identify this species (\( W \)), for which the reference values \( U_{01}, U_{02}, \ldots, U_{0n} \) will be the most appropriate from the plurality of reference species.

Qualitative criteria of species identification

The initial data for the task are measurements \( U_1, U_2, \ldots, U_n \) and a plurality of reference values \( U_{01}, U_{02}, \ldots, U_{0n} \) for each bird species. The mathematical model in the form of a "black box" with the reference parameters, where the input factors are the measurements and the output factor are the identified species is presented in Fig. 1.

\[
\begin{align*}
U_i & \quad \text{Reference parameters, } \quad U^{01}, U^{02}, \ldots, U^{0n} \\
U_{0i} & \quad \text{Type of object}
\end{align*}
\]

Fig. 1. Diagram of the mathematical model.

An algorithm for solving this problem can ensure the selection from variety of bird species the instance \( W^i \), which by its reference indicators \( U_{01}, U_{02}, \ldots, U_{0n} \) most likely corresponds to the measured values. As a first test of conformity between the experimental and reference data we choose the relative sum of squared deviations of sets of component \( U \) and \( U^0 \):

\[
K_i = \sum_{k=1}^{n} f_k \left( \frac{U_k - U_{0k}}{U_{0k}^2} \right)^2, i = 1, 2, \ldots, m. \quad (2)
\]
Coefficient $f_i$ in formula (2) can consider the measurement accuracy of the individual parameter and its contribution to the overall characteristics of the object. The minimum value of the criterion (2) refers to the individuals $W$, the reference parameters of which are consistent with the results of measurements. We represented the results of calculations (2) in the form of a non-decreasing rank

$$K_1 \leq K_2 \leq K_3 \leq \ldots \leq K_n, \quad (3)$$

where the numerical value of the criterion $K_i$ most likely referred to individuals, which is found at this observation, is on the first place.

**RESULTS**

As an example, consider the hypothetical case of processing the results of observations. We have some radar measurements of unknown bird species, like length, wing span, frequency of wingbeats, and airspeed. Reference parameters for certain bird species (Rook, Raven, Mallard, White Stork, Lapwing) are listed in Table 1.

<table>
<thead>
<tr>
<th>Bird species</th>
<th>Length, cm</th>
<th>Wingspan, m</th>
<th>Wingbeat frequency, Hz</th>
<th>Airspeed, km/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rook</td>
<td>60-70</td>
<td>130-140</td>
<td>3-4</td>
<td>50-60</td>
</tr>
<tr>
<td>Raven</td>
<td>60-70</td>
<td>120-130</td>
<td>3-4</td>
<td>40-50</td>
</tr>
<tr>
<td>Duck (Mallard)</td>
<td>57-62</td>
<td>85-95</td>
<td>5-7</td>
<td>72-97</td>
</tr>
<tr>
<td>White Stork</td>
<td>100-115</td>
<td>155-165</td>
<td>1,5-2,5</td>
<td>35-45</td>
</tr>
<tr>
<td>Lapwing</td>
<td>27-33</td>
<td>78-88</td>
<td>40-45</td>
<td>95-105</td>
</tr>
<tr>
<td>Observed species</td>
<td>50-60</td>
<td>80-90</td>
<td>4-6</td>
<td>70-90</td>
</tr>
</tbody>
</table>

Calculation results by formula (2) make it possible to present the rank (3) in the form:

$$K_{\text{maldod}} = 0.058 < K_{\text{rook}} = 1.12 < K_{\text{mula}} = 1.52 < K_{\text{waps}} = 2.91 < K_{\text{stork}} = 4.10, \quad (4)$$

From ranked series (4) it is supposed that Mallard could be the very species regards the observation parameters. Further species, in descending order of probability could be rook, raven, white stork, and lapwing. In some cases, when one of the criteria is in order of magnitude smaller than alternatives, the mathematical data processing can fully meet the needs of the researcher. However, the probability of identifying itself remains unknown. Furthermore, if subsequent values $K_2, K_3$ and others are close enough to $K_i$, it is possible to refer the observable object to species $W_j, W_k, \text{etc}.$

**Bird selection with help of Pearson criteria.**

Let’s formulate the second criterion, which allows to eliminate all species do not agree with measurements with certain degree of reliability from consideration. The deviation value of measured parameter $U_k$ from the reference value $U_0^i k$ for each species is a random variable. We assume that this value is subject to a normal distribution with variance $(\sigma^2)^2$. We introduce the normalized parameter

$$z_k^i = \frac{U_k - U_0^i k}{\sigma_k}, \quad (5)$$

which is also a random variable obeying a normal distribution with zero value and variance equal to one. Let consider the sum of squared deviations for a particular type:

$$\chi_i^2 = \sum_{k=n}^{k=m} \left( z_k^i \right)^2, \quad i=1,2,\ldots,m. \quad (6)$$

Since $z_0, z_0, \ldots, z_0$ are independent random variables with zero expectation and are subject to a normal distribution with a variance equal to 1, the value (6) is subject to the distribution of Pearson $\chi^2$ (chi-square) with $n$ number of degrees of freedom [7]. The solution of the problem is the verification of the null hypothesis $H_0$ about the correspondence between the measured and reference data for the $W^j$ individuals.

$H_0$ hypothesis testing is conducted as follows:

1. On the basis of the reference data $\{U_0^i k\}$ and the measurement results $\{U_k\}$ the Pearson criterion values (6) are calculated for each reference species. Let mark them as $\chi^2\text{meas}$. 

---

**Table 1. Initial data for identification of unknown individuals (Eastwood, 1967; Gania et al., 1991).**
2. From Table 2 the critical values \( \chi^2_{cr} \) are selected. They depend on the number of measured values \( n \) and the significance level \( q \).

3. If the condition \( \chi^2_{meas} < \chi^2_{cr} \) (7) is true, then the hypothesis \( H_0 \) is accepted. Otherwise it is rejected in favor of the alternative hypothesis \( H_1 \), whereby the measurement results do not support the type of detection \( W^3 \) species.

### Table 2. Critical values \( \chi^2_{cr} \) for significance level \( q = 0.1 \) (Kremer, 2003).

<table>
<thead>
<tr>
<th>( N )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^2_{cr} )</td>
<td>2.71</td>
<td>4.60</td>
<td>6.25</td>
<td>7.78</td>
<td>9.24</td>
</tr>
</tbody>
</table>

Testing of null hypothesis eliminates all the options that do not satisfy the condition (7). As a result of the analysis we get one of three possible conclusions: 1. Condition (7) does not true for all elements of the set \( \{ W^3 \} \). Consequently, the measurement results do not meet all bird species from reference database.

2. Condition (7) is true for only one of the bird species with \( \chi^2_{meas} \) criterion value. In this case, the exposition that the measurement results testify the observed individuals belong to \( W^3 \) species is true.

3 Condition (7) is true for two or more \( \chi^2 \) values.

The implementation of first and second cases leads to the clear conclusion on the identification of unknown species. Last case requires further investigation. Calculation of Pearson \( \chi^2_{meas} \) criterion according to the formula (6)

\[
\chi^2_{meas} = \sum_{i=1}^{k} \frac{(P_k - P_{ki}^{0})^2}{\sigma_k^2}
\]

for data from Table 1 gives the following results:

- \( \chi^2_{meas} \) of mallard = 2.3, \( \chi^2_{meas} \) of rook = 10.0, \( \chi^2_{meas} \) of lapwing = 19.0, \( \chi^2_{meas} \) of stock = 26.6

It was assumed that the standard deviation \( \sigma_k \) in the formula (5) was equal to the sum of the absolute measurement error from the difference between the maximum and minimum values in the reference parameters. According to Table 2 the critical Pearson test value for four parameters at a significance level of \( q = 0.1 \) is equal to 7.78. Therefore, the last four individuals in series (8) - Rook, Raven, Lapwing, Stork should be excluded from consideration. By data of Table 3 we could determine the probability of each discussed events and we find that the probability of Mallard identifying at value \( \chi^2_{meas} = 2.3 \) is 0.65, and for the Rook with criterion \( \chi^2_{meas} = 10.0 \), the probability \( R_{meas} \) is 0.05.

### Table 3. The dependence of the probability on the number of measured values \( n \) and \( \chi^2 \) (Kremer, 2003).

<table>
<thead>
<tr>
<th>Number of measured parameters, ( n )</th>
<th>0.95</th>
<th>0.90</th>
<th>0.80</th>
<th>0.70</th>
<th>0.50</th>
<th>0.30</th>
<th>0.20</th>
<th>0.10</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability, ( P )</td>
<td>0.35</td>
<td>0.58</td>
<td>1.00</td>
<td>1.42</td>
<td>2.37</td>
<td>3.66</td>
<td>4.64</td>
<td>6.25</td>
<td>7.82</td>
</tr>
<tr>
<td>4</td>
<td>0.71</td>
<td>1.06</td>
<td>1.65</td>
<td>2.20</td>
<td>3.36</td>
<td>4.88</td>
<td>5.99</td>
<td>7.78</td>
<td>9.84</td>
</tr>
<tr>
<td>5</td>
<td>1.14</td>
<td>1.61</td>
<td>2.34</td>
<td>3.00</td>
<td>4.35</td>
<td>6.06</td>
<td>7.29</td>
<td>9.24</td>
<td>11.7</td>
</tr>
</tbody>
</table>

We should note that the order of bird species in ranks (3) and (8) constructed using the criterion (2) and Pearson criterion (6) coincide. In given example we considered the most favorable situation, when statistical analysis revealed only one species. If study results are not very accurate and Pearson criterion allows several alternatives, it is recommended to turn to the Fisher test.

**The identification of species using Fisher Test.**

Let we selected several bird species to be identified, \( W \), on the basis of the survey results and screening of insignificant variants using Pearson test. The deviation of measurement results from standard \( W^3 \) parameters will be characterized by dispersions

\[ D_1 < D_2 < D_3 \ldots \]  

The value of the corrected variance in normalized form can be represented as:

\[ \sigma_k^2 = \frac{1}{n_k-1} \sum_{k=1}^{k=n} \left( \frac{P_k - P_{ki}^{0}}{\sigma_k} \right)^2 \]  

where \( n_k \) - number of measured parameters for bird species from set of various individuals \( \{ W^3 \} \).

We can formulate the null hypothesis of equality between two dispersions \( D_1 \) and \( D_2 \):

\[ H_0: D_1 = D_2 \]  

when the alternative hypothesis is:

\[ H_1: D_2 > D_1 \]  

F-test is calculated by formula

\[ F_{meas} = D_2 / D_1 \]
for the degrees of freedom, respectively, \( k_1 = n_1 - 1 \) u \( k_2 = n_2 - 1 \).

When \( F_{\text{max}} > F_0 \) is true, the null hypothesis (11) is rejected and the alternative hypothesis is accepted (12). In this case, it can be argued that the results of the measurements allowed identification of the first specimen \( \mathcal{W}_i \). Due to (9) it does not make sense to carry out a comparison of \( D_1 \) and \( D_i \) for \( i > 2 \), because all the criteria (13) will be obviously larger than \( F_0 \).

If \( F_{\text{min}} < F_0 \) is true, the null hypothesis is accepted. Consequently, the measurement conducted in this case, are not accurate enough to determine the bird species. The critical value \( F_0 \) depends on the number of degrees of freedom \( k_1, k_2 \) as well as on chosen significance level, \( q \). The value of \( q \) is accepted equal to 0.05 In many technical applications (Table 3).

Table 3. Dependence of Fisher criterion on degrees of freedom \( k_1 \) and \( k_2 \) for the significance level \( q = 0.05 \) (Kremer, 2003).

<table>
<thead>
<tr>
<th>( k_1 )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>19.0</td>
<td>19.2</td>
<td>19.2</td>
<td>19.3</td>
</tr>
<tr>
<td>3</td>
<td>9.55</td>
<td>9.28</td>
<td>9.12</td>
<td>9.00</td>
</tr>
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<td>4</td>
<td>6.94</td>
<td>6.59</td>
<td>6.39</td>
<td>6.26</td>
</tr>
<tr>
<td>5</td>
<td>5.79</td>
<td>5.41</td>
<td>5.19</td>
<td>5.05</td>
</tr>
</tbody>
</table>

If \( F_{\text{max}} > F_0 (q = 0.05) \), then the alternative hypothesis is true with a high degree of reliability, so there is a risk to reject the correct null hypothesis. In other case, when \( q \) is relatively large, it is more likely to accept the null hypothesis, which is in fact incorrect. The choice is still a researcher task.

We have concluded, that the suggested method could be used in identifying an unknown species of birds from results of the measured parameters of observed unknown individual. The body mass, length, wingspan, flight speed and other bird characteristics could be used as parameters, measured by visual survey. The use of a mathematical model allows to increase the range of observations and automate the processing of the experimental data obtained with the use of modern methods (radiotelemetry, bioacoustics, radar, etc.).

REFERENCES


